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# Mathematics (MP Board)

## SOLUTION

### Question 1

(i) (a) always irrational
(ii) (a) 2
(iii) (b) 3
(iv) (C) 20
(v) (C) −4
(vi) (b) Parallel

### Question 2

(i) Similar
(ii) $\theta/360 \times 2\pi r$
(iii) Arithmetic Progression
(iv) $b^2 - 4ac$
(v) Perpendicular
(vi) $45^\circ$

### Question 3

(i) True
(ii) True
(iii) True

(iv) False
(v) True
(vi) True

## Question 4

(i) Probability of sure event — (c) 1
(ii) Value of tan 30° — (a) $1/\sqrt{3}$
(iii) Value of Sin 0° — (b) 0
(iv) $1 - \sin^2 \theta$ — (f) $\cos^2\theta$
(v) Total surface area of cylinder — (d) $2\pi r(r+h)$
(vi) Total surface area of sphere — (e) $4\pi r^2$

## Question 5

(i) Angle of Depression
(ii) $a_n = a+(n-1)d$
(iii) 8
(iv) $\sqrt{a^2+b^2}$
(v) Infinite
(vi) "If a line is drawn parallel to one side of triangle, and it intersects other two sides; then it divides the other sides in same ratio."

Question 6

Clearly,

Product of the Numbers = HCF × LCM

$\Rightarrow 306 \times 657 = 9 \times LCM$

$\Rightarrow LCM = \frac{306 \times 657}{9}$

$\Rightarrow LCM = 22,338$

Ans

OR

Clearly,

$3 \times 5 \times 7 + 7 = 7(3 \times 5 \times 1 + 1)$

$= 7(16)$

$= 7 \times 2^4$

Clearly by using Fundamentle theorem of Arithmetic we can say that $3 \times 5 \times 7 + 7$ is a composite Number

Question 7

Let,

$a = 3$ ; $d = 6-3 = 3$ and $a_n = 111$

Now,

$$a_n = a + (n-1)d$$

$$\Rightarrow 111 = 3 + (n-1)3$$

$$\Rightarrow \frac{108}{3} = (n-1)$$

$$\Rightarrow n = 36+1$$

$$\Rightarrow n = 37$$

Ans

OR

Clearly,

$12-9 = 3$

$15-12 = 3$

$18-15 = 3$

∵ Difference of each term to its next term is constant it is an AP.

$a_{16} = a + (16-1)d$

$a_{16} = 9 + (16-1)3$

$= 9 + 15 \times 3$

$a_{16} = 54$ Ans

## Question 8

Clearly,

$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$

$= x(x+2) + 3(x+2)$

$= (x+2)(x+3)$

$\Rightarrow x = -2$ or, $x = -3$

Hence $\alpha = -2$ and $\beta = -3$

$\alpha + \beta = -2 + (-3) = -5$ and $\alpha\beta = (-2)(-3) = 6$

$\therefore$ for given polynomial $f(x) = x^2 + 5x + 6$, we have

$-\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{5}{1} = -5$ and, $\frac{\text{constant term}}{\text{coefficient of } x^2} = \frac{6}{1} = 6$

Hence, the relation between zeroes and coefficients is verified.

OR

Clearly,

$\alpha+\beta = -b/a = -2/1 = -2$

and $\alpha\beta = c/a = 3/1 = 3$

$\Rightarrow (\alpha+\beta)^2 - 2\alpha\beta = (-2)^2 - 2(3)$

$= 4-6$

$= -2$

Question 9

Clearly,

$p(x) = k[x^2 - (\alpha+\beta)x + \alpha\beta]$

and given,

$\alpha+\beta = 0$ and $\alpha\beta = \sqrt{5}$

$\Rightarrow p(x) = k[x^2 - (0)x + \sqrt{5}]$

$= k[x^2 + \sqrt{5}]$ Ans

OR

Clearly,

$$p(x) = k\left[x^2 - (\alpha+\beta)x + \alpha\beta\right]$$

and given,

$$\alpha+\beta = 1/4 \quad \text{and} \quad \alpha\beta = -1$$

$$\Rightarrow \quad p(x) = k\left[x^2 - \frac{1}{4}x - 1\right]$$

$$= \left[4x^2 - x - 4\right] \quad \text{Ans}$$

Question 10

Given,

$$2x + 3y = 9$$

$$\Rightarrow \quad 2x = 9 - 3y$$

$$x = \frac{9-3y}{2} \rightarrow (1)$$

Also,

$$3x + 2y = 12 \rightarrow (2)$$

Putting value of $x$ from eqⁿ ① in eqⁿ ②

$$3\left(\frac{9-3y}{2}\right) + 2y = 12$$

$$3(9-3y) + 4y = 24$$

$$27 - 9y + 4y = 24$$

$$5y = 3$$

$$y = 3/5$$

$$\Rightarrow x = \left(\frac{9 - 3(3/5)}{2}\right)$$

$$= \frac{45-9}{10}$$

$$x = \frac{36}{10} = \frac{18}{5}$$

$\therefore y = 3/5$ and $x = 18/5$

OR

Given,

$$x + 2y - 4 = 0$$

and $$2x + 4y - 12 = 0$$

Clearly,

$$\frac{1}{2} = \frac{2}{4} \neq \frac{4}{12}$$

$\therefore$ $$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$ for $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$

$\Rightarrow$ The lines of given equation are parallel

$\therefore$ The two tracks don't cross each other.

Question 11

We know that,

Minute hand complete one round of $360°$ in 60 minutes

$\therefore$ Angle formed for 1 min $= \frac{360}{60} = 6°$

$\therefore$ Area swept in 5 min $= 5 \times \frac{6}{360} \times \pi \times (14)^2$

[$\because r = 14$ and Area of sector $= \frac{\theta}{360} \times \pi r^2$]

$= 5 \times \frac{1}{60} \times \frac{22}{7} \times 14 \times 14$

$= 51.34 \text{ cm}^2$ (Approx)

OR

We know that,

for a Quadrant angle subtended by arc at center is $90°$

Given,

Circumference $= 22$ cm

$\Rightarrow 2\pi r = 22$

$\Rightarrow 2 \times \frac{22}{7} \times r = 22$

$\Rightarrow r = \frac{7}{2}$

$\therefore$ Area of Quadrant $= \frac{90}{360} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ [$\because$ Area $= \frac{\theta}{360} \pi r^2$]

$= 9.625\ cm^2$

Question 12

Clearly,

in $\Delta ABC$ and $\Delta PQR$

$\angle B = \angle Q = 90^\circ$ [Pole and Tower]

$\angle C = \angle R = \theta$ [elevation of sun]

$\Rightarrow \Delta ABC \sim \Delta PQR$ [By AA Similarity]

$\Rightarrow \frac{AB}{BC} = \frac{PQ}{QR}$ [By CPST]

$\Rightarrow \frac{6}{4} = \frac{h}{28}$

$\Rightarrow h = 42m$

Ans

OR

Given :- DE ∥ BC ; AE = 1cm , AD = 1.5cm and DB = 3cm

To find :- EC

Sol^n :- Using BPT,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = 2cm$$

Ans

Question 13

Required Point is P(0, y)

∴ PA = PB

Using Distance formula $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$, we obtain

$\therefore \sqrt{(0-6)^2+(y-5)^2} = \sqrt{(0+4)^2+(y-3)^2}$

$\Rightarrow 36+y^2+25-10y = 16+y^2-6y+9$

$\Rightarrow 61-25 = 10y-6y$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$ Ans

OR

Clearly, $\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$ [Using distance formula $D = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$]

$\Rightarrow x^2-6x+9+y^2-12y+36 = x^2+6x+9+y^2-8y+16$

$\Rightarrow -12x-4y = -20$

$\Rightarrow 3x+y = 5$ Ans

Question 14

Given,

$$\sin A = 3/4$$

Now,

$$\cos A = \sqrt{\cos^2 A}$$

$$= \sqrt{1 - \sin^2 A}$$

$$= \sqrt{1 - (3/4)^2}$$

$$= \sqrt{\frac{16-9}{16}}$$

$$\cos A = \frac{\sqrt{7}}{4}$$

and

$$\tan A = \frac{\sin A}{\cos A}$$

$$= \frac{3/4}{\sqrt{7}/4} = \frac{3\sqrt{7}}{7}$$ Ans

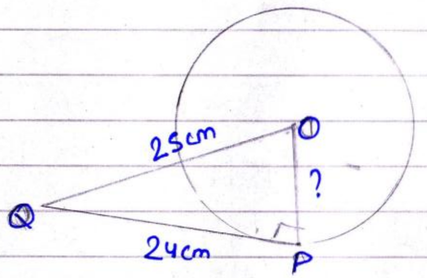
OR

$\sin 60 \cos 30 + \sin 30 \cos 60 = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

Question 15




Given :- length of tangent PQ = 24cm
and Distance from center OQ = 25cm

To find :- OP = ?

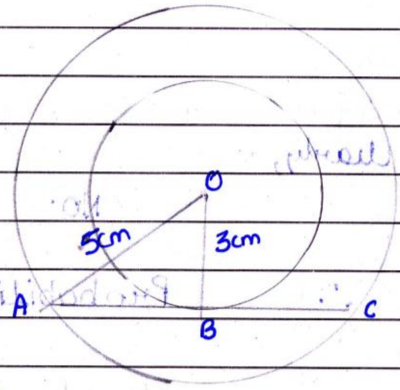
Sol$^n$ :- ∴ Tangent is perpendicular to radius at point of contact

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow 25^2 = OP^2 + 24^2$

$\Rightarrow OP = 7cm.$

OR




Given :- Two concentric circles of radius 5cm and 3cm respectively.

To find :- AC = ?

Sol$^n$ :- clearly, AB acts like a tangent to the circle with radius 3cm

$\Rightarrow$ $\angle B = 90°$ [ tangents are perpendicular to radius at point of contact.

$\therefore$ Using PGT,

$$AO^2 = OB^2 + AB^2$$

$$\Rightarrow 5^2 = 3^2 + AB^2$$

$$\Rightarrow AB = 4cm$$

$\because$ $OB \perp AC$, Therefore OB bisects AC

$\therefore$ $AC = 2AB = 8cm.$

Question 16

let,

Event of getting a 'red' card ($E_1$) = 26
Event of getting a queen ($E_2$) = 4

∴ Event of getting a Red card or a queen(E) = 26 + 4 − 2
= 28

∴ P(E) = $\frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$

= $\frac{28}{52}$

= $\frac{7}{13}$.

∴ P(E') = 1 − P(E) = 1 − $\frac{7}{13}$ = $\frac{6}{13}$ Ans

OR

(i) Event of getting a number greater than 4 ($E_1$) = 2

P($E_1$) = $\frac{\text{No. of favourable outcomes}}{\text{Total No. of outcomes}}$ = $\frac{2}{6}$ = $\frac{1}{3}$

(ii) Let $E_2$ be the event of getting a number less than 4

Number of favourable outcomes = 3

Total number of possible outcomes = 6

∴ Probability of getting a number less than 4 is

$$P(E_2) = \frac{3}{6} = \frac{1}{2}$$

Question 17

let,

E = Event of getting a perfect Square Number

= { 1, 4, 9, 16, 25, 36, 49, 64, 81 }

Now,

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{Total No. of Outcomes}}$$

$$= \frac{9}{90}$$

$$\frac{1}{10}$$

OR

let

Event of getting a Red ball (E) = 3

∴ $P(E) = \frac{\text{No. of favourable outcome}}{\text{Total No. of outcomes}}$

$= \frac{3}{3+2+5}$

$= \frac{3}{10}$

∴ $P(E') = 1 - \frac{3}{10} = \frac{7}{10}$

Question 18

let,

Breath of given rectangle is $y$

∴ length $= 2y + 1$

$\Rightarrow (2y+1) \cdot y = 528$

$\Rightarrow 2y^2 + y - 528 = 0$

$\Rightarrow 2y^2 + 33y - 32y - 528 = 0$

$\Rightarrow y(2y+33) - 16(2y+33) = 0$

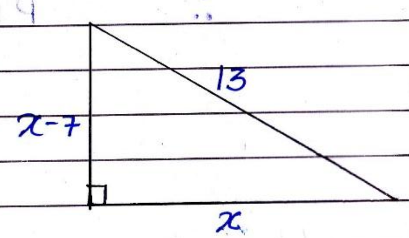
$\Rightarrow (y-16)(2y+33) = 0$

either $y = 16$ or $y = -\frac{33}{2}$ [ -ve Not Possible

$\therefore$ $y = 16m$ , and

length $= 2\times16+1 = 33\,m$

OR




Clearly

Using PGT, we get
$(x-7)^2 + x^2 = 13^2$

$\Rightarrow x^2 - 14x + 49 + x^2 = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

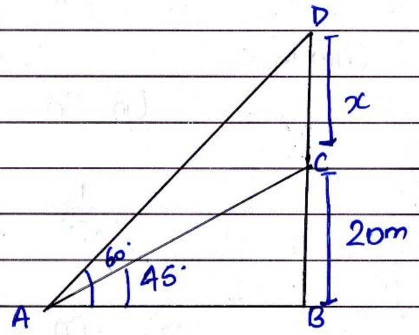
$\Rightarrow (x+5)(x-12) = 0$

$\Rightarrow$ either $x = -5$ or $x = 12$ cm.

We will consider positive value

$\therefore$ Altitude is $12-7 = 5$ cm and the base is 12 cm.

Question 19




Clearly,

$\tan 45 = \frac{BC}{AB}$ #

$\Rightarrow 1 = \frac{20}{AB}$

$\Rightarrow AB = 20$m

$\therefore \tan 60 = \frac{BD}{AB}$

$$\Rightarrow \sqrt{3} = \frac{20+x}{20}$$

$$\Rightarrow 20+x = 20\sqrt{3}$$

$$\Rightarrow x = 20\sqrt{3} - 20$$

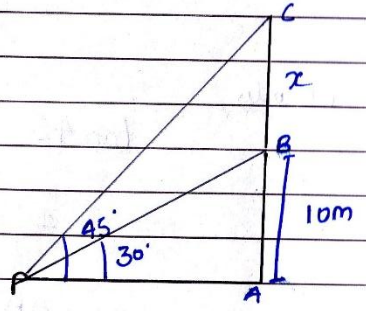
$$\Rightarrow x = 20(\sqrt{3}-1)\ m$$

Ans

∴ Height of tower is $20(\sqrt{3}-1)$ m.

OR

Question 19




Clearly,

$$\tan 30 = \frac{AB}{PA}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10}{PA}$$

$$\Rightarrow PA = 10\sqrt{3} m.$$

∴ Distance of building from point P is $10\sqrt{3}$ m.

Now,

$$\tan 45^\circ = \frac{AC}{AP}$$

$$\Rightarrow 1 = \frac{x+10}{10\sqrt{3}}$$

$$\Rightarrow x+10 = 10\sqrt{3}$$

$$\Rightarrow x = 10\sqrt{3} - 10$$

$$\Rightarrow x = 10(\sqrt{3}-1) = 10(1.732-1) = 10(0.732) = 7.32\,m$$

$\therefore$ Height of flag staff is is 7.32 m.

Question 20

let,

p and q are two co-prime integers and let us assume that $6+\sqrt{2}$ be a rational Number

$$\Rightarrow 6+\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \sqrt{2} = \frac{p}{q} - 6$$

But it's given that, $\sqrt{2}$ is an irrational number

$\Rightarrow$ $\frac{P - 6q}{q}$ is also irrational

This contradicts our assumption that $6+\sqrt{2}$ is rational

$\therefore$ $6+\sqrt{2}$ is an irrational number.

OR

Clearly,

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$35 = 5 \times 7$

LCM $= 2^3 \times 3^2 \times 5 \times 7$

$= 2520$

HCF $= \underline{\underline{1}}$

Question 21

let,

Req. number be $x$

$\therefore \quad x^2 - 84 = 3(x+8)$

$\Rightarrow \quad x^2 - 3x - 24 - 84 = 0$

$\Rightarrow \quad x^2 - 12x + 9x - 108 = 0$

$\Rightarrow \quad x(x-12) + 9(x-12) = 0$

$\Rightarrow \quad (x+9)(x-12) = 0$

either $x = -9$ or $x = 12$

But Req. number is a natural number

$\therefore \quad x = 12$ Ans

or

let,

Required number be $x$

$\therefore (x)^2 + (x+2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x + 4 = 290$

$\Rightarrow 2x^2 + 4x - 286 = 0$

$\Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0$

$\Rightarrow x(x+13) - 11(x+13) = 0$

$\Rightarrow (x-11)(x+13) = 0$

either $x = 11$ or $x = -13$

But Required number is a positive Number

$\therefore x = 11$ Ans

and $x + 2 = 11 + 2 = 13$

Question 22

Given,

Cuboid with length $(l)$ = 15cm ; breath $(b)$ = 10cm and Height $(h)$ = 3·5cm and a right circular conical depression of radius $(r)$ = 0·5cm and depression $(h_c)$ = 1·4cm

$\therefore$ Volume of wood = Volume of cuboid − 4× Volume of each depression

$= l \times b \times h - 4 \times \frac{1}{3}\pi r^2 h_c$

$= 15 \times 10 \times 3.5 - 4 \times \frac{1}{3} \times \frac{22}{7} \times (0.5)^2 \times 1.4$

$= 525 - 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{1}{4} \times \frac{14}{10}$

$= 525 - \frac{1}{3} \times 22 \times \frac{2}{10}$

$= 525 - 1.466$

$= 523.534\ cm^3$ (approx)

OR

Given,

Solid cylinder of Radius $(r) = 1.4/2 = 0.7 cm$

Height $(h) = 2.4 cm$ and a Conical cavity of

Height $(h') = 2.4 cm$ and Radius $(r') = 0.7 cm$

TSA of Req. Solid = $CSA_{cylinder} + CSA_{cone} + Area_{base}$

$= 2\pi rh + \pi rl + \pi r^2$ $\left[\because l = \sqrt{(h')^2 + (r')^2}\right]$

$= \pi r \left[2h + \sqrt{(r')^2 + (h')^2} + r\right]$

$= \frac{22}{7} \times 0.7 \left[2\cdot(2.4) + \sqrt{(0.7)^2 + (2.4)^2} + 0.7\right)$

$= 2.2 \left[4.8 + \sqrt{6.25} + 0.7\right]$

$= 2.2 \left[4.8 + 2.5 + 0.7\right]$

$= 2.2 \times 8$

$= 17.6 \ cm^2$

Question 23

| | Family Size | No. of families ($f_i$) | ($x_i$) | $f_i x_i$ |
|---|---|---|---|---|
| i | 1-3 | 7 | 2 | 14 |
| ii | 3-5 | 8 | 4 | 32 |
| iii | 5-7 | 2 | 6 | 12 |
| iv | 7-9 | 2 | 8 | 16 |
| v | 9-11 | 1 | 10 | 10 |

$\therefore \Sigma f_i = 20$

$\Rightarrow \Sigma f_i x_i = 84$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{84}{20}$$

$$= 4.2$$

$\therefore$ Mean of above data = 4.2

OR

Clearly,

Highest frequency of given data is 12

∴ Modal class is 200 − 250

⇒ lower limit of modal class ($l$) = 200
frequency of modal class ($f_1$) = 12
frequency of preceding class ($f_0$) = 5
frequency of succeeding class ($f_2$) = 2
Class size ($h$) = 250 − 200 = 50

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - f_0 - f_2} \times h$$

$$= 200 + \frac{12-5}{2\times12 - 5 - 2} \times 50$$

$$= 200 + \frac{7}{17} \times 50$$

$$= 200 + 20.588$$

Mode = 220.59 [Approx].